

LISTA 3 - RADICIAÇÃO E POTENCIAÇÃO

OBS.: É PRECISO FAZER ALGUNS EXERCÍCIOS.

5) RAÍZ QUADRADA DE $16/9$

SOLUÇÃO

$$\sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{\sqrt{4^2}}{\sqrt{3^2}} = \frac{4}{3}$$

6) RAÍZ CÚBICA DE $-27/8$

SOLUÇÃO

$$\sqrt[3]{\frac{-27}{8}} = \frac{\sqrt[3]{-27}}{\sqrt[3]{8}} = \frac{\sqrt[3]{(-3)^3}}{\sqrt[3]{2^3}} = \frac{-3}{2} = -\frac{3}{2}$$

CALCULE A EXPRESSÃO SEM USAR UMA CALCULADORA

10. $\sqrt[3]{216}$

FATORANDO (DECOMPOSIÇÃO EM N.º PRIMOS)

216 | 2)
108 | 2)
54 | 2)
27 | 3)
9 | 3)
3 | 3)
1

~~216 = 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 2 \cdot 2~~
~~= 2^3 \cdot 3^3~~

$$216 = 2^3 \cdot 3^3$$

~~$\sqrt[3]{216} = \sqrt[3]{2 \cdot 3 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2}$~~

$$\sqrt[3]{216} = \sqrt[3]{2^3 \cdot 3^3} = \sqrt[3]{2^3} \cdot \sqrt[3]{3^3} = 2 \cdot 3 = 6$$

$\therefore \sqrt[3]{216} = 6$

$$\textcircled{11} \sqrt[3]{\frac{-64}{27}} = \frac{\sqrt[3]{-64}}{\sqrt[3]{27}} = \frac{\sqrt[3]{-64}}{\sqrt[3]{27}} = \frac{\sqrt[3]{(-4)^3}}{\sqrt[3]{3^3}} = \frac{-4}{3}$$

SIMPLIFIQUE REMOVENDO FATORES DO RADICANDO.

DICA: P/ OS N^{OS} DEVEMOS DECOMPOR OS M^{BR}OS EM FATORES PRIMOS.

$$23. \sqrt{288} = \sqrt{2^5 \cdot 3^2} = \sqrt{2^5} \cdot \sqrt{3^2} = \sqrt{2^2 \cdot 2^2 \cdot 2} \cdot 3 = \sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{2} \cdot 3 = 2 \cdot 2 \cdot 3 \sqrt{2} = 12\sqrt{2}$$

288	2
144	2
72	2
36	2
18	2
9	3
3	3
1	

$$288 = 2^5 \cdot 3^2$$

$$\therefore \sqrt{288} = 12\sqrt{2}$$

$$25. \sqrt[3]{-250} = \sqrt[3]{-(5^3 \cdot 2)} = \sqrt[3]{-1 \cdot (5^3 \cdot 2)} = \sqrt[3]{-1} \cdot \sqrt[3]{5^3} \cdot \sqrt[3]{2} = -1 \cdot 5 \sqrt[3]{2} = -5\sqrt[3]{2}$$

250	5
50	5
10	2
5	5
1	

$$250 = 5^3 \cdot 2$$

$$\therefore \sqrt[3]{-250} = -5\sqrt[3]{2}$$

$$28. \sqrt[3]{-27x^3y^6} = \sqrt[3]{-1} \cdot \sqrt[3]{3^3 x^3 y^3 y^3} = -1 \cdot \sqrt[3]{3^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{y^3} \cdot \sqrt[3]{y^3} = -1 \cdot 3 \cdot x \cdot y \cdot y = -3xy^2$$

$$\therefore \sqrt[3]{-27x^3y^6} = -3xy^2$$

3

(32) $\sqrt{108x^4y^9} = \sqrt{2^2 \cdot 3^2 \cdot 3 \cdot x^2 \cdot x^2 \cdot y^2 \cdot y^2 \cdot y^2 \cdot y^2 \cdot y} = \sqrt{2^2} \sqrt{3^2} \sqrt{3} \sqrt{x^2} \sqrt{x^2} \sqrt{y^2} \sqrt{y^2} \sqrt{y^2} \sqrt{y^2} \sqrt{y}$
 $= 2 \cdot 3 \cdot \sqrt{3} |x| \cdot |x| \cdot |y| \cdot |y| \cdot |y| \cdot |y| \cdot \sqrt{y} = 6x^2y^4\sqrt{3y}$
 $= 6x^2y^4\sqrt{3y}$

SOLUÇÃO

108	2
54	2
27	3
9	3
3	3
1	

$108 = 2^2 \cdot 3^2 \cdot 3$

$\therefore \sqrt{108x^4y^9} = 6x^2y^4\sqrt{3y}$

RACIONALIZE O DENOMINADOR

33. $\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

$\therefore \frac{4}{\sqrt{2}} = 2\sqrt{2}$

35. $\frac{1}{\sqrt{x^2}} = \frac{1}{\sqrt{x^2}} \cdot \frac{\sqrt{x^3}}{\sqrt{x^3}} = \frac{\sqrt{x^3}}{\sqrt{x^2} \cdot \sqrt{x^3}} = \frac{\sqrt{x^3}}{\sqrt{x^2 x^3}} = \frac{\sqrt{x^3}}{\sqrt{x^5}} = \frac{\sqrt{x^3}}{|x|}$

$\therefore \frac{1}{\sqrt{x^2}} = \frac{\sqrt{x^3}}{|x|}$

(36)

$$\begin{aligned} \frac{2}{\sqrt[4]{y}} &= \frac{2}{\sqrt[4]{y}} \cdot \frac{\sqrt[4]{y^{4-1}}}{\sqrt[4]{y^{4-1}}} = \frac{2 \sqrt[4]{y^3}}{\sqrt[4]{y} \cdot \sqrt[4]{y^3}} = \frac{2 \sqrt[4]{y^3}}{\sqrt[4]{y \cdot y^3}} = \\ &= \frac{2 \sqrt[4]{y^3}}{\sqrt[4]{y^4}} = \frac{2 \sqrt[4]{y^3}}{|y|} \end{aligned}$$

$$\therefore \frac{2}{\sqrt[4]{y}} = \frac{2 \sqrt[4]{y^3}}{|y|}$$

CONVERTA PARA A FORMA RADICAL

$$43. a^{3/4} b^{1/4} = \sqrt[4]{a^3} \cdot \sqrt[4]{b} = \sqrt[4]{a^3 b}$$

$$44. x^{2/3} y^{1/3} = \sqrt[3]{x^2} \cdot \sqrt[3]{y} = \sqrt[3]{x^2 y}$$

$$45. x^{-5/3} = \frac{1}{x^{5/3}} = \frac{1}{\sqrt[3]{x^5}}$$

$$46. (xy)^{-3/4} = \frac{1}{(xy)^{3/4}} = \frac{1}{\sqrt[4]{(xy)^3}} = \frac{1}{\sqrt[4]{x^3 y^3}}$$

ESCREVA USANDO UM RADICAL SIMPLES

$$47. \sqrt{\sqrt{2x}} = \sqrt{(2x)^{1/2}} = \left((2x)^{1/2} \right)^{1/2} = (2x)^{1/4} = \sqrt[4]{2x}$$

OU

$$\sqrt{\sqrt{2x}} = \sqrt[2 \cdot 2]{2x} = \sqrt[4]{2x}$$

$$(48) \sqrt{\sqrt[3]{3x^2}} = \sqrt[2 \cdot 3]{3x^2} = \sqrt[6]{3x^2}$$

$$(51) \frac{\sqrt[5]{a^2}}{\sqrt[3]{a}} = \frac{a^{2/5}}{a^{1/3}} = a^{2/5 - 1/3} = a^{6/15 - 5/15} = a^{1/15} = \sqrt[15]{a}$$

$$(52) \sqrt{a} \sqrt[3]{a^2} = a^{1/2} \cdot a^{2/3} = a^{1/2 + 2/3} = a^{3/6 + 4/6} = a^{7/6} = \sqrt[6]{a^7}$$

SIMPLIFIQUE AS EXPRESSÕES EXPONENCIAIS

$$(53) \frac{a^{3/5} a^{1/3}}{a^{3/2}} = a^{3/5 + 1/3 - 3/2} = a^{-17/30}$$

$$\frac{3}{5} + \frac{1}{3} - \frac{3}{2} = \frac{6 \cdot 3 + 10 - 45}{30} = \frac{18 + 10 - 45}{30} = \frac{28 - 45}{30} = -\frac{17}{30}$$

$$\therefore \frac{a^{3/5} a^{1/3}}{a^{3/2}} = a^{-17/30}$$

$$(55) (a^{5/3} b^{3/4}) (3a^{1/3} b^{5/4}) = 3a^{5/3 + 1/3} b^{3/4 + 5/4} = 3a^{6/3} b^{8/4} = 3a^2 b^2$$

$$\therefore (a^{5/3} b^{3/4}) (3a^{1/3} b^{5/4}) = 3a^2 b^2$$

$$\begin{aligned}
 (58) \quad \frac{(p^2 q^4)^{1/2}}{(27 q^3 p^6)^{1/3}} &= \frac{p^{2 \cdot \frac{1}{2}} q^{4 \cdot \frac{1}{2}}}{27^{1/3} q^{3 \cdot \frac{1}{3}} p^{6 \cdot \frac{1}{3}}} = \frac{p q^2}{\sqrt[3]{27} q p^2} = \\
 &= \frac{p q^2 \cdot q^{-1} p^{-2}}{3} = \frac{p^{1-2} q^{2-1}}{3} = \frac{p^{-1} q}{3} = \frac{q}{3p}
 \end{aligned}$$

SIMPLIFIQUE AS EXPRESSÕES RADICAIS

$$\begin{aligned}
 (61) \quad \sqrt{9 x^{-6} y^4} &= \sqrt{9} \sqrt{x^{-6}} \sqrt{y^4} = 3 \sqrt{\frac{1}{x^6}} \sqrt{y^2 \cdot y^2} = \\
 &= 3 \frac{\sqrt{1}}{\sqrt{x^6}} \cdot \sqrt{y^2} \sqrt{y^2} = \frac{3 |y| |y|}{\sqrt{x^2 \cdot x^2 \cdot x^2}} = \frac{3 y^2}{\sqrt{x^2} \sqrt{x^2} \sqrt{x^2}} = \frac{3 y^2}{|x| |x| |x|} = \\
 &= \frac{3 y^2}{x^2 |x|}
 \end{aligned}$$

$$\therefore \sqrt{9 x^{-6} y^4} = \frac{3 y^2}{x^2 |x|}$$

$$\begin{aligned}
 (65) \quad \sqrt[3]{\frac{4x^2}{y^2}} \cdot \sqrt[3]{\frac{2x^2}{y}} &= \sqrt[3]{\frac{4x^2}{y^2} \cdot \frac{2x^2}{y}} = \sqrt[3]{\frac{8x^4}{y^3}} = \frac{\sqrt[3]{8} \sqrt[3]{x^3 \cdot x}}{\sqrt[3]{y^3}} = \frac{2 \sqrt[3]{x^3} \sqrt[3]{x}}{y} \\
 &= \frac{2x \sqrt[3]{x}}{y}
 \end{aligned}$$

$$\begin{aligned}
 (70) \quad \sqrt{18x^2y} + \sqrt{2y^3} &= \sqrt{2 \cdot 9x^2y} + \sqrt{2y^2y} = \sqrt{9} \sqrt{x^2} \sqrt{2y} + \sqrt{y^2} \sqrt{2y} = \\
 &= 3|x| \sqrt{2y} + |y| \sqrt{2y} = (3|x| + |y|) \sqrt{2y}
 \end{aligned}$$

$$\therefore \sqrt{18x^2y} + \sqrt{2y^3} = (3|x| + |y|) \sqrt{2y}$$

71) $\sqrt{2+6} \circ \sqrt{2} + \sqrt{6}$

$\sqrt{2+6} = \sqrt{8} = \sqrt{2^3} = \sqrt{2^2 \cdot 2} = 2\sqrt{2}$

$\sqrt{2} + \sqrt{6} = \sqrt{2} + \sqrt{2 \cdot 3} = \sqrt{2} + \sqrt{2} \cdot \sqrt{3} = \sqrt{2}(1 + \sqrt{3})$

NOTE QUE

$(1 + \sqrt{3}) > 2$ ENTÃO $(1 + \sqrt{3})\sqrt{2} > 2\sqrt{2}$

$\therefore \sqrt{2+6} < \sqrt{2} + \sqrt{6}$

73) $(3^{-2})^{-1/2} \circ 3$

$\therefore (3^{-2})^{-1/2} = 3$

$(3^{-2})^{-1/2} = 3^{-2 \cdot -1/2} = 3^1 = 3$

75) $\sqrt[4]{(-2)^4} \circ -2$

$\therefore \sqrt[4]{(-2)^4} > -2$

$\sqrt[4]{(-2)^4} = |-2| = 2$

76) $\sqrt[3]{(-2)^3} \circ -2$

$\therefore -2 = -2$

$\sqrt[3]{(-2)^3} = -2$