Problems

SETS AND SUBSETS

1.2 List the elements of each set where $\mathbf{N} = \{1, 2, 3, \ldots\}$.

(*a*) $A = \{x \in \mathbf{N} \mid 3 < x < 9\}$

(*b*) $B = \{x \in \mathbf{N} \mid x \text{ is even}, x < 11\}$

(*c*) $C = \{x \in \mathbf{N} \mid 4 + x = 3\}$

1.3 Let $A = \{2, 3, 4, 5\}$.

(*a*) Show that A is not a subset of $B = \{x \in \mathbf{N} \mid x \text{ is even}\}$.

(*b*) Show that A is a proper subset of $C = \{1, 2, 3, \ldots, 8, 9\}$.

SET OPERATIONS

1.4 Let $\mathbf{U} = \{1,2, \ldots, 9\}$ be the universal set, and let

$$A = \{1, 2, 3, 4, 5\}, \quad C = \{5, 6, 7, 8, 9\}, \quad E = \{2, 4, 6, 8\},$$
$$B = \{4, 5, 6, 7\}, \quad D = \{1, 3, 5, 7, 9\}, \quad F = \{1, 5, 9\}.$$

Find: (*a*) $A \cup B$ and $A \cap B$; (*b*) $A \cup C$ and $A \cap C$; (*c*) $D \cup F$ and $D \cap F$.

Recall that the union $X \cup Y$ consists of those elements in either X or Y (or both), and that the intersection $X \cap Y$ consists of those elements in both X and Y.

1.5 Consider the sets in the preceding Problem 1.4. Find:

(*a*) A^C, B^C, D^C, E^C; (*b*) $A \setminus B, B \setminus A, D \setminus E$; (*c*) $A \oplus B,\ C \oplus D,\ E \oplus F$.

Recall that:

(1) The complements X^C consists of those elements in $\mathbf{U}$ which do not belong to X.

(2) The difference $X \setminus Y$ consists of the elements in X which do not belong to Y.

(3) The symmetric difference $X \oplus Y$ consists of the elements in X or in Y but not in both.

1.6 Show that we can have: (*a*) $A \cap B = A \cap C$ without $B = C$; (*b*) $A \cup B = A \cup C$ without $B = C$.

VENN DIAGRAMS

1.13 Determine the validity of the following argument:

S_1: All my friends are musicians.

S_2: John is my friend.

S_3: None of my neighbors are musicians.

S : John is not my neighbor.

1.15 In a survey of 120 people, it was found that:

65 read *Newsweek* magazine, 20 read both *Newsweek* and *Time*,
45 read *Time*, 25 read both *Newsweek* and *Fortune*,
42 read *Fortune*, 15 read both *Time* and *Fortune*,
8 read all three magazines.

(a) Find the number of people who read at least one of the three magazines.

(b) Fill in the correct number of people in each of the eight regions of the Venn diagram in Fig. 1-9(a) where N, T, and F denote the set of people who read *Newsweek*, *Time*, and *Fortune*, respectively.

(c) Find the number of people who read exactly one magazine.

Supplementary Problems

SETS AND SUBSETS

1.26 Which of the following sets are equal?

$A = \{x \mid x^2 - 4x + 3 = 0\}$, $C = \{x \mid x \in \mathbf{N}, x < 3\}$, $E = \{1, 2\}$, $G = \{3, 1\}$,
$B = \{x \mid x^2 - 3x + 2 = 0\}$, $D = \{x \mid x \in \mathbf{N}, x \text{ is odd}, x < 5\}$, $F = \{1, 2, 1\}$, $H = \{1, 1, 3\}$.

1.27 List the elements of the following sets if the universal set is $\mathbf{U} = \{a, b, c, \ldots, y, z\}$.

Furthermore, identify which of the sets, if any, are equal.

$A = \{x \mid x \text{ is a vowel}\}$, $C = \{x \mid x \text{ precedes } f \text{ in the alphabet}\}$,
$B = \{x \mid x \text{ is a letter in the word "little"}\}$, $D = \{x \mid x \text{ is a letter in the word "title"}\}$.

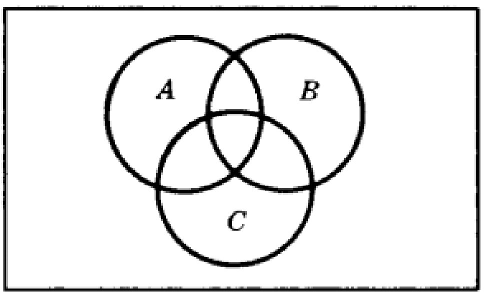
1.28 Let $A = \{1, 2, \ldots, 8, 9\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 3, 5, 7, 9\}$, $D = \{3, 4, 5\}$, $E = \{3, 5\}$.
Which of the these sets can equal a set X under each of the following conditions?

(a) X and B are disjoint. (c) $X \subseteq A$ but $X \not\subseteq C$.
(b) $X \subseteq D$ but $X \not\subseteq B$. (d) $X \subseteq C$ but $X \not\subseteq A$.

VENN DIAGRAMS

1.34 The Venn diagram in Fig. 1-5(a) shows sets A, B, C. Shade the following sets:

(a) $A \backslash (B \cup C)$; (b) $A^{C} \cap (B \cup C)$; (c) $A^{C} \cap (C \backslash B)$.



(a)

Fig. 1-5

1.36 Consider the following assumptions:

S_1: All dictionaries are useful.

S_2: Mary owns only romance novels.

S_3: No romance novel is useful.

Use a Venn diagram to determine the validity of each of the following conclusions:

(a) Romance novels are not dictionaries.

(b) Mary does not own a dictionary.

(c) All useful books are dictionaries.

ALGEBRA OF SETS AND DUALITY

1.38 Use the laws in Table 1-1 to prove each set identity:

(a) $(A \cap B) \cup (A \cap B^C) = A$

(b) $A \cup B = (A \cap B^C) \cup (A^C \cap B) \cup (A \cap B)$

Table 1-1 Laws of the algebra of sets

Idempotent laws:	(1a) $A \cup A = A$	(1b) $A \cap A = A$
Associative laws:	(2a) $(A \cup B) \cup C = A \cup (B \cup C)$	(2b) $(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws:	(3a) $A \cup B = B \cup A$	(3b) $A \cap B = B \cap A$
Distributive laws:	(4a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	(4b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws:	(5a) $A \cup \emptyset = A$	(5b) $A \cap \mathbf{U} = A$
	(6a) $A \cup \mathbf{U} = \mathbf{U}$	(6b) $A \cap \emptyset = \emptyset$
Involution laws:	(7) $(A^C)^C = A$	
Complement laws:	(8a) $A \cup A^C = \mathbf{U}$	(8b) $A \cap A^C = \emptyset$
	(9a) $\mathbf{U}^C = \emptyset$	(9b) $\emptyset^C = \mathbf{U}$
DeMorgan's laws:	(10a) $(A \cup B)^C = A^C \cap B^C$	(10b) $(A \cap B)^C = A^C \cup B^C$

FINITE SETS AND THE COUNTING PRINCIPLE

1.39 Determine which of the following sets are finite:

(a) Lines parallel to the x axis.

(b) Letters in the English alphabet.

(c) Integers which are multiples of 5.

(d) Animals living on the earth.

1.41 A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of three popular options, air-conditioning (A), radio (R), and power windows (W), were already installed. The survey found:

15 had air-conditioning (A), 12 had radio (R), 11 had power windows (W), 5 had A and P, 9 had A and R, 4 had R and W, 3 had all three options.

Find the number of cars that had: (a) only W; (b) only A; (c) only R; (d) R and W but not A; (e) A and R but not W (f) only one of the options; (g) at least one option; (h) none of the options.