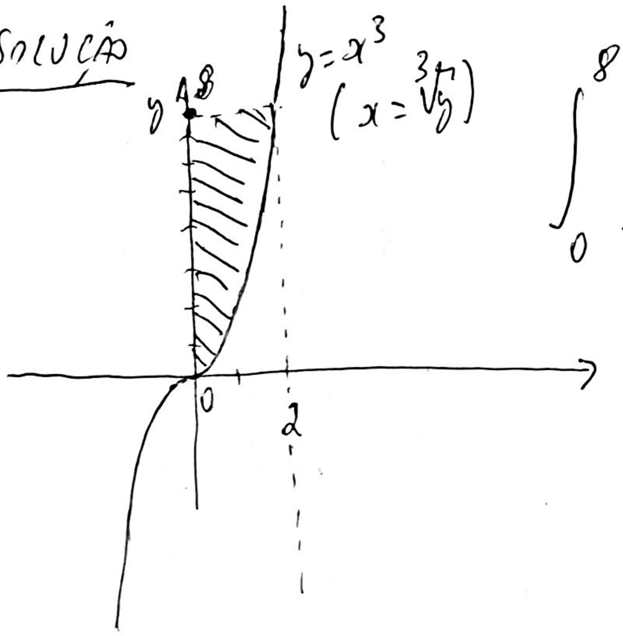
ESBOCE A REGIÃO DE INTEGRAÇÃO E MUDE A ORDEM DE INTEGRAÇÃO 1

(a) $\int_0^2 \int_{x^3}^8 f(x,y)\, dy\, dx$

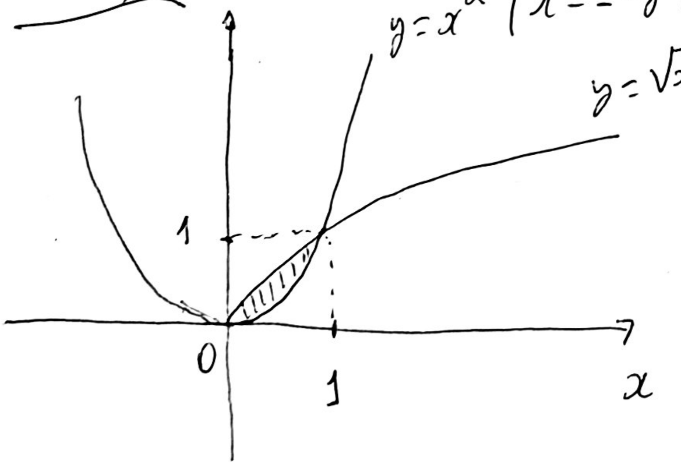
SOLUÇÃO



$$\int_0^8 \int_0^{\sqrt[3]{y}} f(x,y)\, dx\, dy$$

(b) $\int_0^1 \int_{x^2}^{\sqrt{x}} f(x,y)\, dy\, dx$

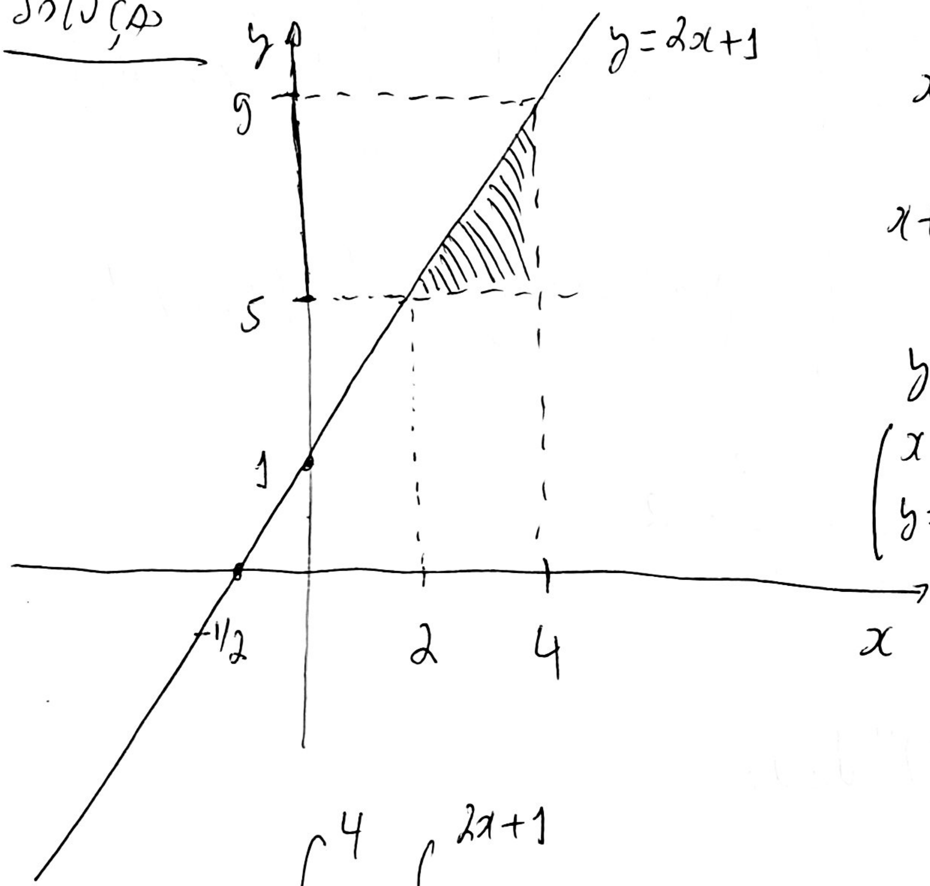
SOLUÇÃO



$$\int_0^1 \int_{y^2}^{\sqrt{y}} f(x,y)\, dx\, dy$$

(C) $$\int_5^9 \int_{\frac{y}{2}-\frac{1}{2}}^4 f(x,y)\,dx\,dy$$

Solução



$$x = \frac{y}{2} - \frac{1}{2} \Rightarrow$$

$$x + \frac{1}{2} = \frac{y}{2} \Rightarrow y = 2\left(x + \frac{1}{2}\right) \Rightarrow$$

$$y = 2x + 1$$

$$\left(\begin{array}{l} x = 0 \Rightarrow y = 1 \\ y = 0 \Rightarrow x = -\frac{1}{2} \end{array}\right)$$

$$\int_2^4 \int_5^{2x+1} f(x,y)\,dy\,dx$$

CALCULE A INTEGRAL ITERADA

(a) $\displaystyle\int_0^2 \int_0^{\frac{\pi}{2}} x \cos y \, dy \, dx$

SOLUÇÃO

$$\int_0^2 \int_0^{\frac{\pi}{2}} x \cos y \, dy \, dx = \int_0^2 \left(\int_0^{\frac{\pi}{2}} x \cos y \, dy \right) dx = \int_0^2 x \left(\int_0^{\frac{\pi}{2}} \cos y \, dy \right) dx =$$

$$= \int_0^2 x \left(\operatorname{sen} y \Big|_0^{\frac{\pi}{2}} \right) dx = \int_0^2 x \left(\operatorname{sen} \frac{\pi}{2} - \operatorname{sen} 0 \right) dx = \int_0^2 x (1 - 0) \, dx =$$

$$= \int_0^2 x \, dx = \frac{x^2}{2} \Big|_0^2 = \frac{2^2}{2} - \frac{0^2}{2} = 2$$

(b) $\displaystyle\int_0^1 \int_0^1 v(u - v^2)^4 \, du \, dv$

SOLUÇÃO

$$* \left| \begin{array}{l} x = u - v^2, \\ dx = du \end{array} \right. \qquad \int_0^1 \left(\int_0^1 v (u - v^2)^4 du \right) dv = \int_0^1 v \left(\int_0^1 (u - v^2)^4 du \right) dv$$

$$\int (u - v^2)^4 du \overset{*}{=} \int x^4 dx = \frac{x^5}{5} = \frac{(u - v^2)^5}{5}, \text{ logo, } \int_0^1 (u - v^2)^4 du = \frac{(u - v^2)^5}{5} \Big|_0^1 =$$

$$= \frac{(1 - v^2)^5}{5} - \frac{(0 - v^2)^5}{5} = \frac{(1 - v^2)^5 + v^{10}}{5}$$

PORTANTO,

$$\int_0^1 v\left(\int_0^1 (u-v^2)^4\,du\right)dv = \int_0^1 \frac{v(1-v^2)^5 + v^{10}}{5}\,dv =$$

$$= \int_0^1 \frac{v(1-v^2)^5}{5}\,dv + \int_0^1 \frac{v^{10}}{5}\,dv = \int_0^1 \frac{v(1-v^2)^5}{5}\,dv + \frac{v^{11}}{55}\Big|_0^1 =$$

$$\int_0^1 \frac{v(1-v^2)^5}{5}\,dv + \frac{1}{55}$$

AGORA,

$$\int \frac{v(1-v^2)^5}{5}\,dv \overset{*}{=} \int -\frac{1}{2}x^5\,dx = -\frac{1}{2}\cdot\frac{x^6}{6}$$

$$* \begin{cases} x = 1 - v^2 \\ dx = -2v\,dv \\ v\,dv = -\frac{1}{2}dx \end{cases}$$

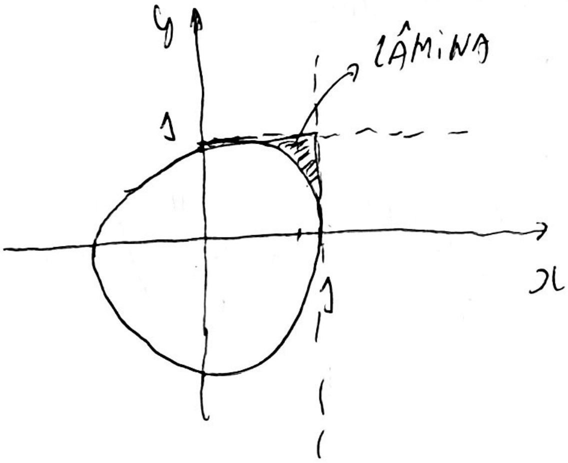
DAÍ, $$\int_0^1 \frac{v(1-v^2)^5}{5}\,dv = -\frac{x^6}{12}\Big|_0^1 = -\frac{(1-v^2)^6}{12}\Big|_0^1 =$$

$$= -\frac{(1-1^2)^6}{12} + \frac{(1-0^2)^6}{12} = \frac{1}{12}$$

$$\therefore \int_0^1\int_0^1 v(u-v^2)^4\,dv = \frac{1}{12} + \frac{1}{55}$$

CÊNTRO DE MASSA DA LÂMINA DADA PELA REGIÃO DO PLANO LIMITADA PELAS CURVAS $x^2+y^2=1$, $x=1$, $y=1$, SENDO $\rho(x,y)=xy$.

SOLUÇÃO



$$m=\iint_D \rho(x,y)\,dA$$

$$m=\int_0^1\left(\int_{\sqrt{1-x^2}}^1 xy\,dy\right)dx =$$

$$=\int_0^1 x\left(\int_{\sqrt{1-x^2}}^1 y\,dy\right)dx=\int_0^1 x\left(\frac{y^2}{2}\Big|_{\sqrt{1-x^2}}^1\right)dx=\int_0^1 x\left(\frac{1}{2}-\frac{(\sqrt{1-x^2})^2}{2}\right)dx=$$

$$=\int_0^1 x\left(\frac{1}{2}-\frac{1+x^2}{2}\right)dx=\int_0^1 x\left(\frac{1-1+x^2}{2}\right)dx=\int_0^1\frac{x^3}{2}dx=\frac{x^4}{8}\Big|_0^1=\frac{1}{8}$$

$$\boxed{\therefore\ m=\frac{1}{8}}$$

JÁ CALCULADA

$$\bar{x}=\frac{1}{m}\iint_D x\,\rho(x,y)\,dA=\frac{1}{\frac{1}{8}}\int_0^1\int_{\sqrt{1-x^2}}^1 x\,xy\,dy\,dx=8\int_0^1 x^2\left(\int_{\sqrt{1-x^2}}^1 y\,dy\right)dx$$

$$=8\int_0^1 x^2\left(\frac{x^2}{2}\right)dx=8\int_0^1\frac{x^4}{2}dx=4\left(\frac{x^5}{5}\Big|_0^1\right)=\frac{4}{5}\ ;\ \bar{x}=\frac{4}{5}$$

$$\bar{y} = \frac{1}{m}\iint_D y\,\rho(x,y)\,dA = 8\int_0^1\int_{\sqrt{1-x^2}}^1 y\,xy\,dy\,dx = 8\int_0^1\int_{\sqrt{1-x^2}}^1 x y^2\,dy\,dx =$$

$$= 8\int_0^1 x\left(\int_{\sqrt{1-x^2}}^1 y^2\,dy\right)dx = 8\int_0^1 x\left(\frac{y^3}{3}\Big|_{\sqrt{1-x^2}}^1\right)dx =$$

$$8\int_0^1 x\left(\frac{1}{3} - \frac{(\sqrt{1-x^2})^3}{3}\right)dx = 8\left(\int_0^1 \frac{x}{3}\,dx - \int_0^1 x\frac{(\sqrt{1-x^2})^3}{3}\,dx\right) =$$

$$= 8\,\frac{x^2}{6}\Big|_0^1 - 8\int_0^1 x\frac{(1-x^2)^{3/2}}{3}\,dx = \frac{8}{6} - 8\int_0^1 x\frac{(1-x^2)^{3/2}}{3}\,dx \quad (1)$$

* CALCULAR A $\int x\frac{(1-x^2)^{3/2}}{3}\,dx = \frac{1}{3}\int x(1-x^2)^{3/2}\,dx$

$$\begin{cases} u = 1-x^2 \\ du = -2x\,dx \\ -\frac{1}{2}du = x\,dx \end{cases}$$

$$\frac{1}{3}\int -\frac{1}{2}u^{3/2}\,du = -\frac{1}{6}\int u^{3/2}\,du = -\frac{1}{6}\,\frac{u^{3/2+1}}{\frac{3}{2}+1} =$$

$$= -\frac{1}{6}\,\frac{u^{5/2}}{\frac{5}{2}} = -\frac{1}{6}\cdot\frac{2}{5}u^{5/2} = \boxed{-\frac{1}{15}(1-x^2)^{5/2}}$$

CONTINUANDO (1) TEMOS

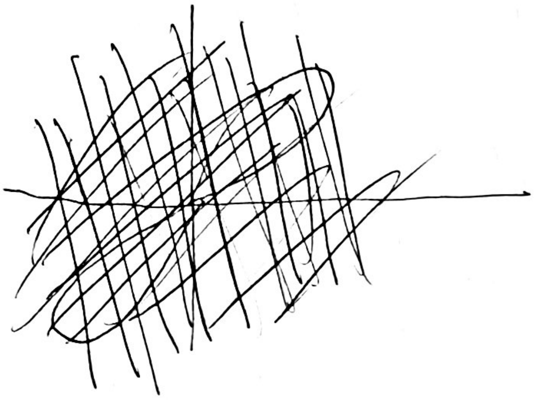
$$\frac{8}{6} - 8\left(-\frac{1}{15}(1-x^2)^{5/2}\Big|_0^1\right) = \frac{8}{6} + \frac{8}{15}\left(\underbrace{(1-1^2)^{5/2}}_{0} - (1-0^2)^{5/2}\right) = \frac{8}{6} + \frac{8}{15}(-1) =$$

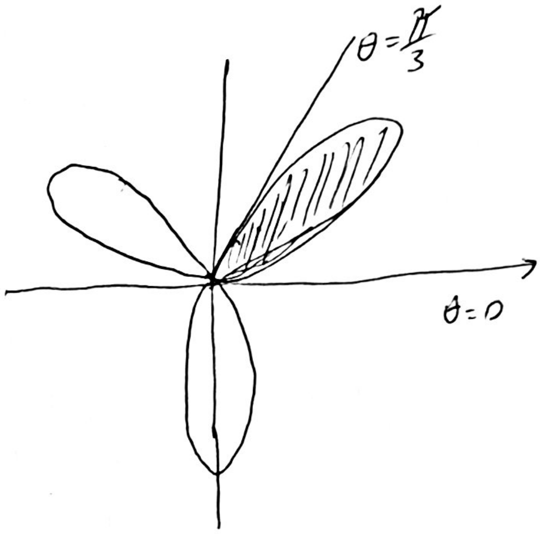
$$= \frac{8}{6} - \frac{8}{15} = \frac{40-16}{30} = \frac{24}{30}$$

$$\bar{y} = \frac{24}{30}$$

$$\boxed{\therefore (\bar{x},\bar{y}) = \left(\frac{4}{5}, \frac{24}{30}\right)}$$

CALCULE A ÁREA DA ROSÁCEA SENDO $r = \operatorname{sen} 3\theta$





$$\int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r,\theta)\, r\, dr\, d\theta$$
R

COMO QUEREMOS CALCULAR A ÁREA $f(r,\theta) = 1$, DAÍ

$$\int_0^{\frac{\pi}{3}} \int_0^{\operatorname{sen} 3\theta} r\, dr\, d\theta = \int_0^{\pi/3} \left(\int_0^{\operatorname{sen} 3\theta} r\, dr \right) d\theta = \int_0^{\frac{\pi}{3}} \left(\frac{r^2}{2} \Big|_0^{\operatorname{sen} 3\theta} \right) d\theta =$$

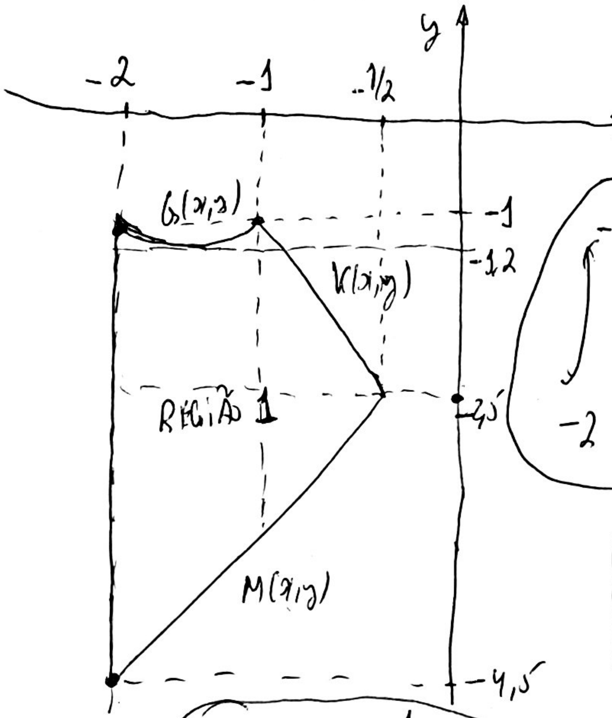
$$= \int_0^{\frac{\pi}{3}} \frac{(\operatorname{sen} 3\theta)^2}{2}\, d\theta = \frac{1}{2} \int_0^{\frac{\pi}{3}} \frac{1 - \cos 6\theta}{2}\, d\theta = \frac{1}{2} \int_0^{\frac{\pi}{3}} \frac{1}{2}\, d\theta - \frac{1}{2} \int_0^{\frac{\pi}{3}} \frac{\cos 6\theta}{2}\, d\theta =$$

$$= \frac{1}{4}\, \theta \Big|_0^{\frac{\pi}{3}} - \frac{1}{4} \frac{\operatorname{sen} 6\theta}{6} \Big|_0^{\frac{\pi}{3}} = \frac{1}{4} \cdot \frac{\pi}{3} - \frac{1}{4} \frac{\overbrace{\operatorname{sen}\left(6 \frac{\pi}{3}\right)}^{0}}{6} = \frac{\pi}{12} - \frac{0}{24} = \frac{\pi}{12}$$

$$\boxed{\therefore \text{ÁREA DA ROSÁCEA } 3 \cdot \frac{\pi}{12} = \frac{\pi}{4}}$$

FATO: $\operatorname{sen}^2 x = \frac{1 - \cos(2x)}{2}$ $\left(\text{Logo, } \operatorname{sen}^2 3\theta = \frac{1 - \cos(2 \cdot 3\theta)}{2} = \frac{1 - \cos 6\theta}{2} \right)$

DETERMINE A INTEGRAL DUPLA NA ORDEM $dx\,dy$ E $dy\,dx$



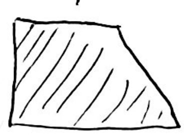
$$\int_{-2}^{-1}\int_{M(x,y)}^{G(x,y)} dy\,dx + \int_{-1}^{-1/2}\int_{M(x,y)}^{K(x,y)} dy\,dx$$

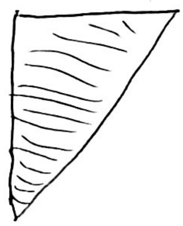
OU

(2 REGIÕES)

OU

$$2\int_{-1,2}^{-1}\int_{G^{-1}(x,y)}^{K^{-1}(x,y)} dx\,dy + \int_{-2,5}^{-1,2}\int_{-2}^{K^{-1}(x,y)} dx\,dy + \int_{-4,5}^{-2,5}\int_{-2}^{M^{-1}(x,y)} dx\,dy$$





(3 REGIÕES)