Exercícios sobre integrais duplas

1–2 Determine $\int_0^5 f(x, y)\, dx$ e $\int_0^1 f(x, y)\, dy$.

1. $f(x, y) = 12x^2y^3$

2. $f(x, y) = y + xe^y$

3–14 Calcule a integral iterada.

3. $\int_1^4 \int_0^2 (6x^2 - 2x)\, dy\, dx$

4. $\int_0^1 \int_1^2 (4x^3 - 9x^2y^2)\, dy\, dx$

5. $\int_0^2 \int_0^{\pi/2} x \operatorname{sen} y\, dy\, dx$

6. $\int_{\pi/6}^{\pi/2} \int_{-1}^5 \cos y\, dx\, dy$

7. $\int_{-3}^3 \int_0^{\pi/2} (y + y^2 \cos x)\, dx\, dy$

8. $\int_0^1 \int_1^2 \dfrac{xe^x}{y}\, dy\, dx$

9. $\int_1^4 \int_1^2 \left(\dfrac{x}{y} + \dfrac{y}{x}\right) dy\, dx$

10. $\int_0^1 \int_0^3 e^{x+3y}\, dx\, dy$

11. $\int_0^1 \int_0^1 v(u - v^2)^4\, du\, dv$

12. $\int_0^1 \int_0^1 xy\sqrt{x^2 + y^2}\, dy\, dx$

13. $\int_0^2 \int_0^{\pi} r \operatorname{sen}^2\theta\, d\theta\, dr$

14. $\int_0^1 \int_0^1 \sqrt{s + t}\, ds\, dt$

15–22 Calcule a integral dupla.

15. $\iint_R \operatorname{sen}(x + y)\, dA$, $R = \{(x, y) \mid 0 \leqslant x \leqslant \pi/2, 0 \leqslant y \leqslant \pi/2\}$

16. $\iint_R (y + xy^{-2})\, dA$, $R = \{(x, y) \mid 0 \leqslant x \leqslant 2, 1 \leqslant y \leqslant 2\}$

17. $\iint_R \dfrac{xy^2}{x^2 + 1}\, dA$, $R = \{(x, y) \mid 0 \leqslant x \leqslant 1, -3 \leqslant y \leqslant 3\}$

15.3 Exercícios

1. $\int_0^4 \int_0^{\sqrt{y}} xy^2\, dx\, dy$

2. $\int_0^1 \int_{2x}^2 (x - y)\, dy\, dx$

3. $\int_0^1 \int_{x^2}^{x} (1 + 2y)\, dy\, dx$

4. $\int_0^2 \int_y^{2y} xy\, dx\, dy$

5. $\int_0^1 \int_0^{s^2} \cos(s^3)\, dt\, ds$

6. $\int_0^1 \int_0^v \sqrt{1 - v^2}\, du\, dv$

7–10 Calcule a integral dupla.

7. $\displaystyle\iint_D y^2\,dA, \quad D = \{(x, y) \mid -1 \leqslant y \leqslant 1, -y - 2 \leqslant x \leqslant y\}$

8. $\displaystyle\iint_D \frac{y}{x^5 + 1}\,dA, \quad D = \{(x, y) \mid 0 \leqslant x \leqslant 1, 0 \leqslant y \leqslant x^2\}$

9. $\displaystyle\iint_D x\,dA, \quad D = \{(x, y) \mid 0 \leqslant x \leqslant \pi, 0 \leqslant y \leqslant \operatorname{sen} x\}$

10. $\displaystyle\iint_D x^3\,dA, \quad D = \{(x, y) \mid 1 \leqslant x \leqslant e,\ 0 \leqslant y \leqslant \ln x\}$

15–16 Defina as integrais iteradas para ambas as ordens de integração. Então, calcule a integral dupla usando a ordem mais fácil e explique por que ela é mais fácil.

15. $\displaystyle\iint_D y\,dA$, D é limitada por $y = x - 2, x = y^2$

16. $\displaystyle\iint_D y^2 e^{xy}\,dA$, D é limitada por $y = x, y = 4, x = 0$

43–48 Esboce a região de integração e mude a ordem de integração.

43. $\displaystyle\int_0^1 \int_0^y f(x, y)\,dy\,dx$

44. $\displaystyle\int_0^2 \int_{x^2}^4 f(x, y)\,dy\,dx$

45. $\displaystyle\int_0^{\pi/2} \int_0^{\cos x} f(x, y)\,dy\,dx$

46. $\displaystyle\int_{-2}^2 \int_0^{\sqrt{4-2}} f(x, y)\,dx\,dy$

47. $\displaystyle\int_1^2 \int_0^{\ln x} f(x, y)\,dy\,dx$

48. $\displaystyle\int_0^1 \int_{\operatorname{arctg} x}^{\pi/4} f(x, y)\,dy\,dx$

55–56 Expresse D como a união de regiões do tipo I ou do tipo II e calcule a integral.

55. $\displaystyle\iint_D x^2\,dA$

52. $\displaystyle\iint_D y\,dA$